

# Multidimensional Network Analysis

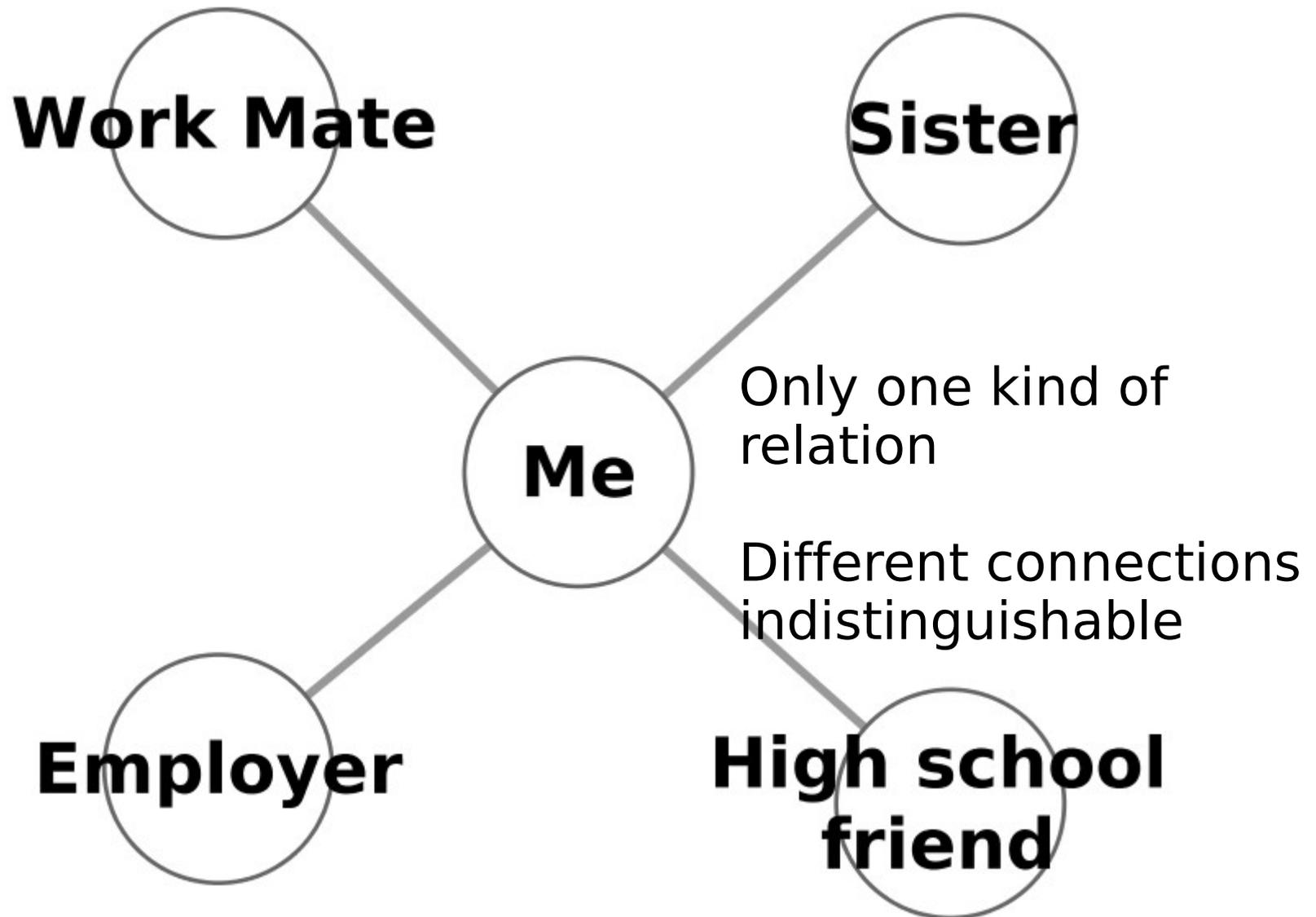
PhD Seminar – Unipi

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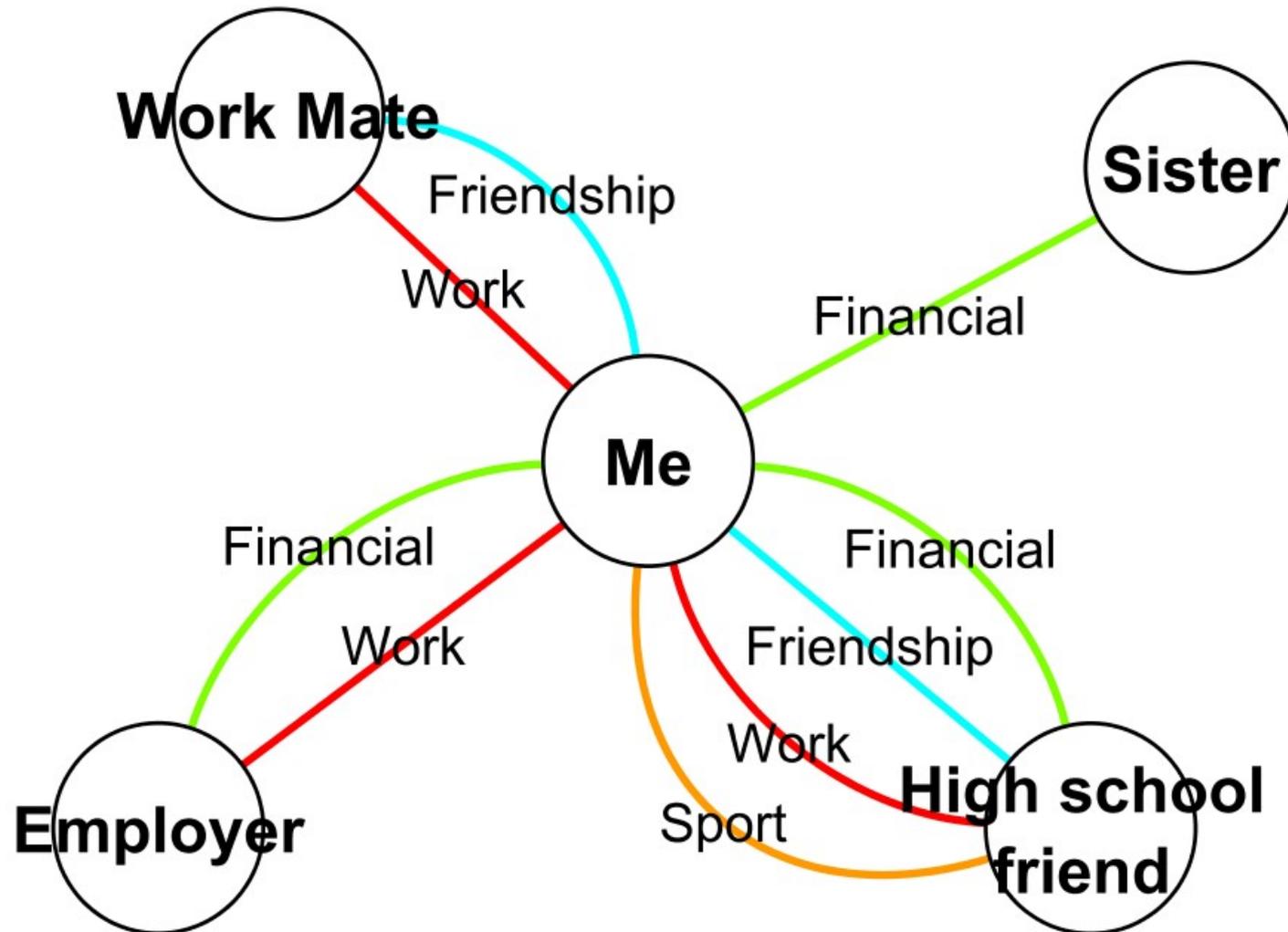
# Classical Network Representation



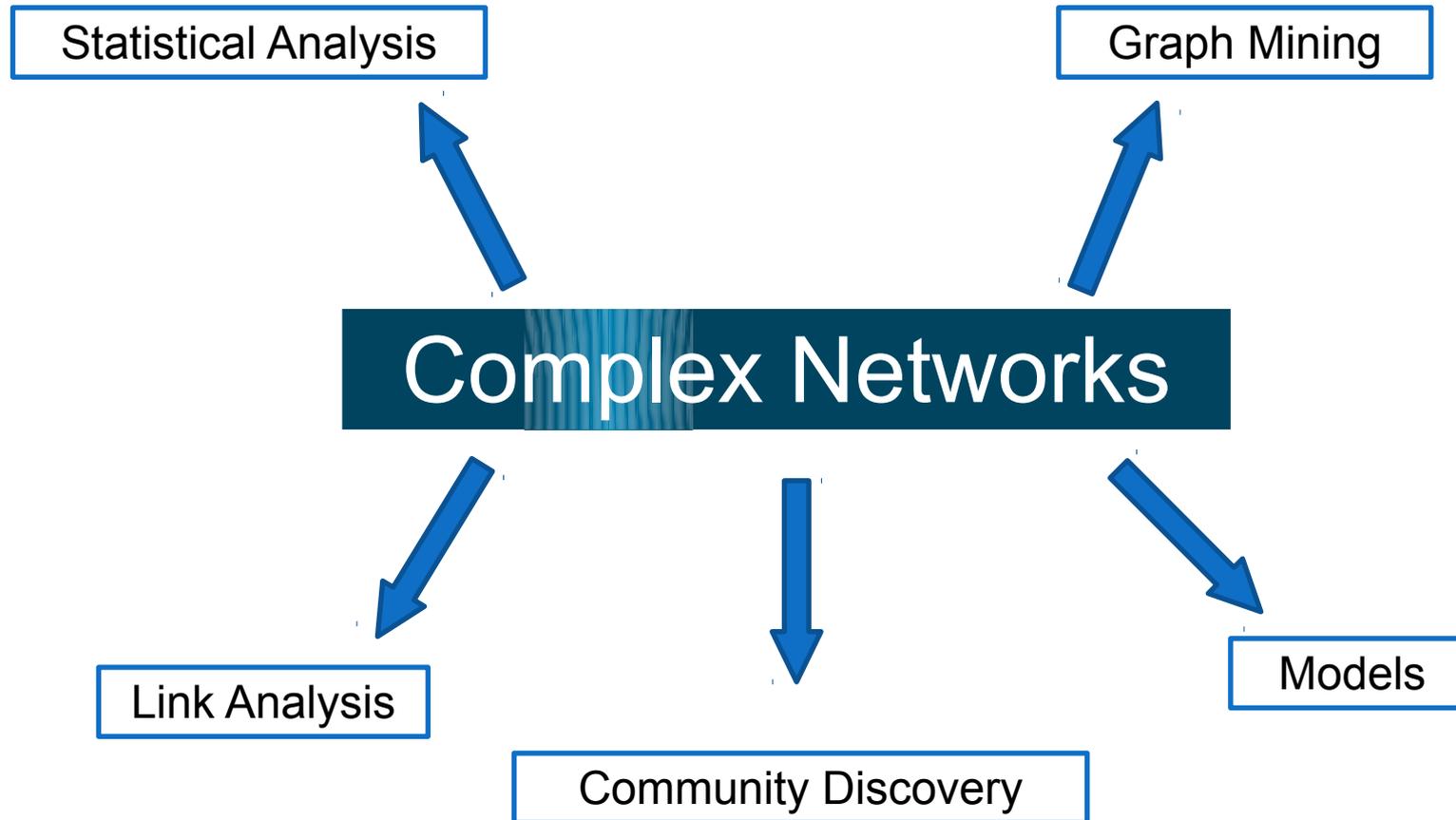
# Real world examples of Multidimensionality

- Relations in different Social Networks (different qualities)
  - Friendship in **facebook**
  - Work in **LinkedIn**
  - Music in **last.fm**
  - Photography in **flickr**
- Citations about different topics (different qualities)
- Voting relation in different years (different quantities)

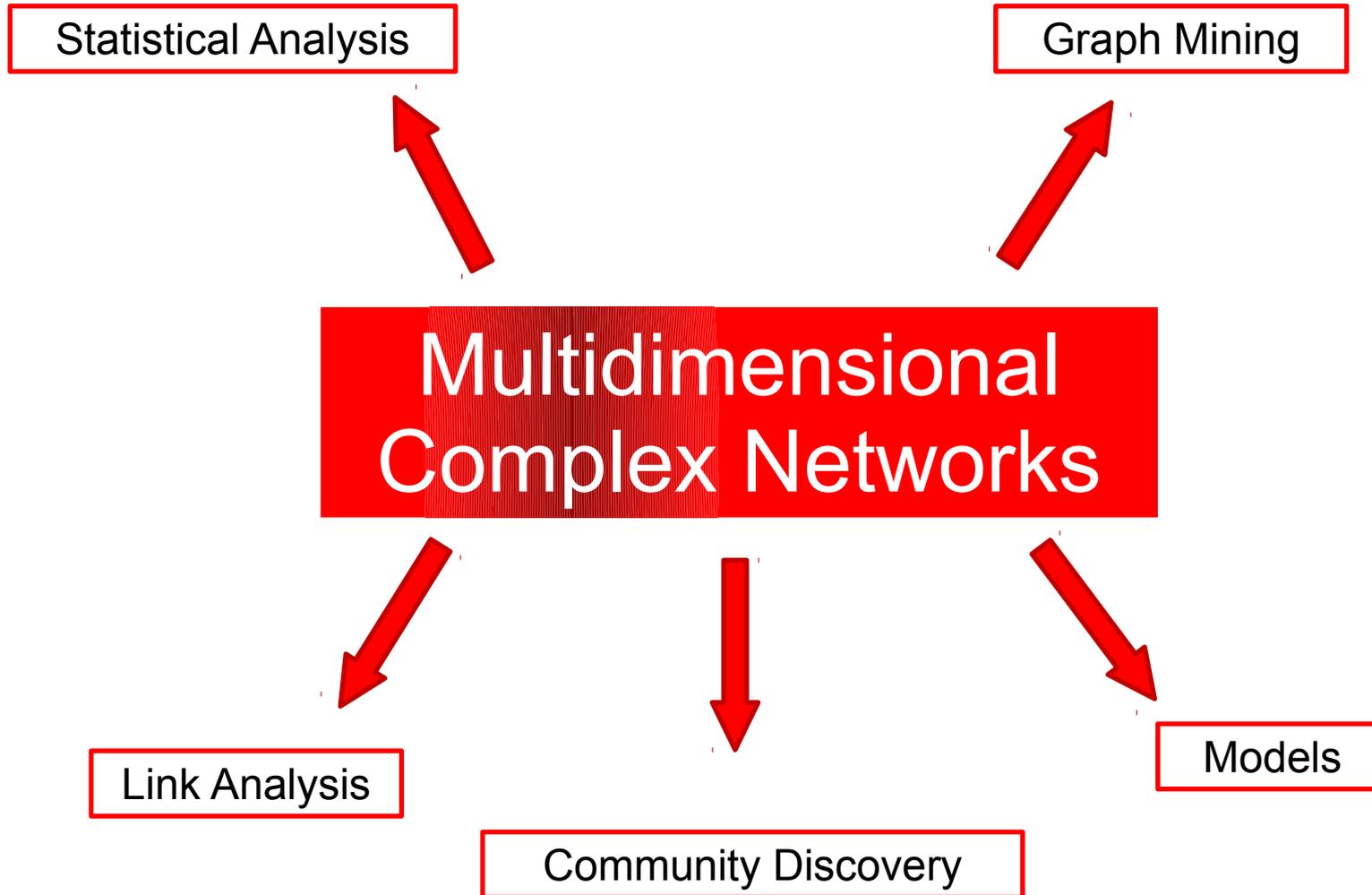
# A possible model



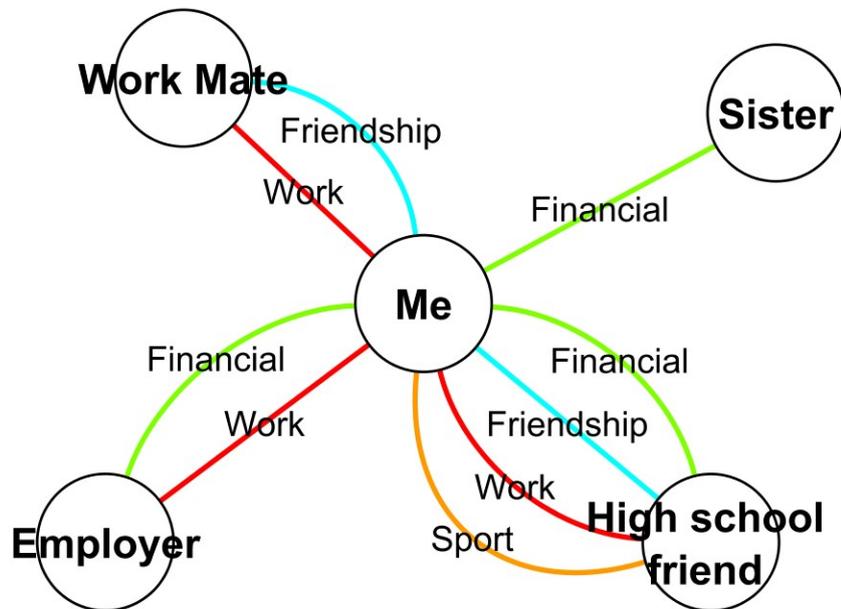
# A New World Arises



# A New World Arises



# Focusing on the basis



$$G = (V, E, L)$$

$$\textit{Neighbors}(v) \neq \textit{Degree}(v)$$

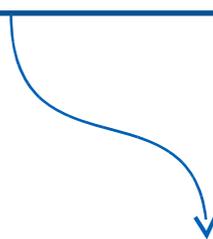
$$\textit{Neighbors}_{XOR}(v, D) =$$

$$|\{u \in V \mid \exists d \in D : (u, v, d) \in E \wedge \nexists d' \notin D : (u, v, d') \in E\}|$$

# Dimension Relevance

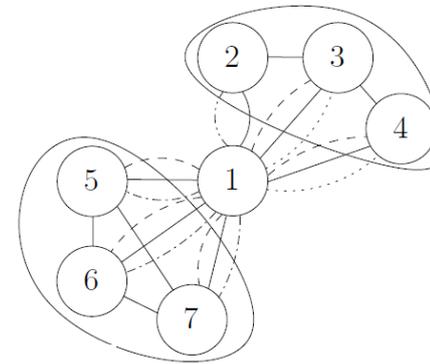
$$DimRelevance(v, D) = \frac{Neighbors(v, D)}{Neighbors(v, L)}$$

$$DimRelevance_{XOR}(v, D) = \frac{Neighbors_{XOR}(v, D)}{Neighbors(v, L)}$$

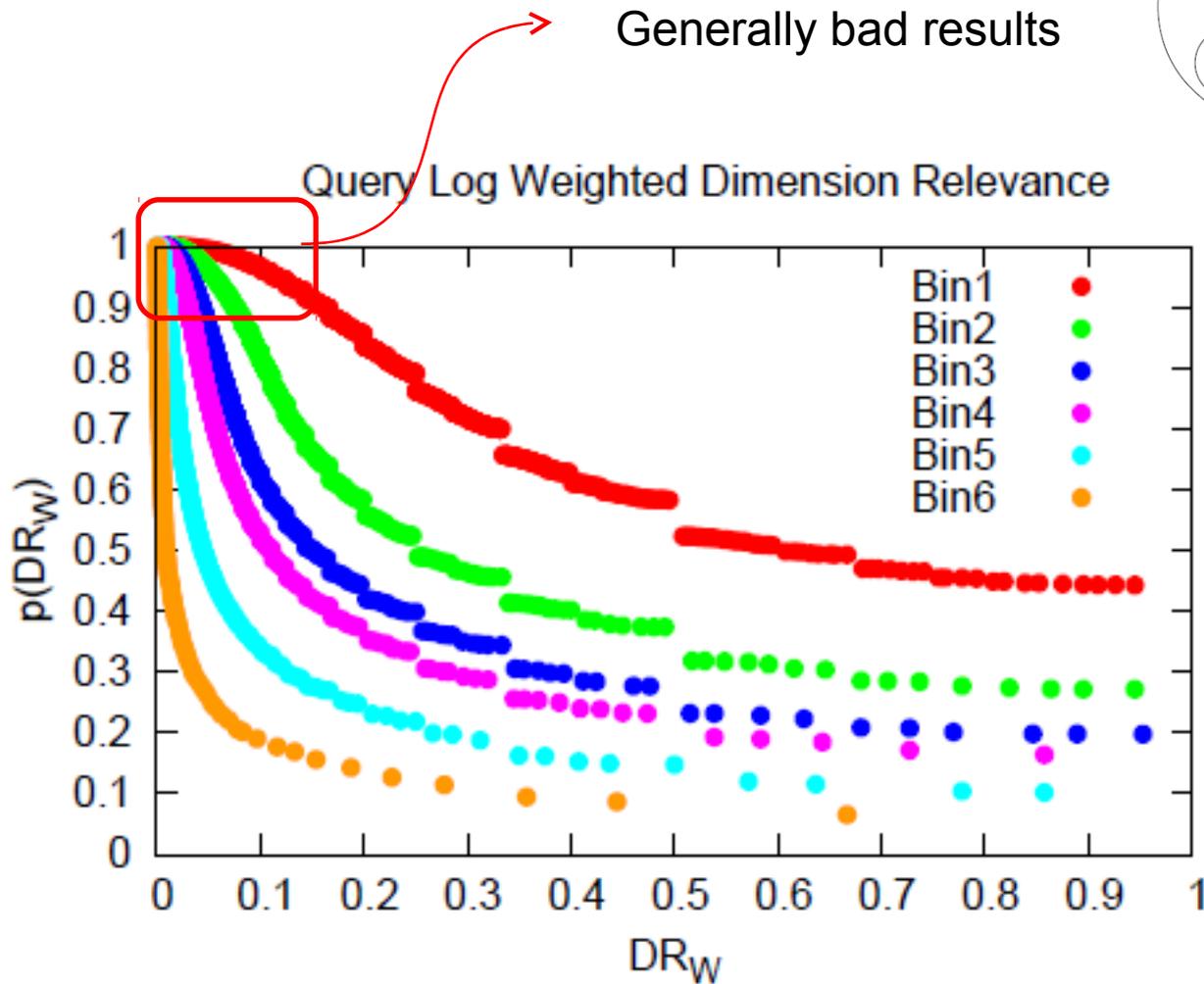

$$DimRelevance_W(v, D) = \frac{\sum_{u \in NeighborSet(v, D)} \frac{n_{uvd}}{n_{uv}}}{Neighbors(v, L)}$$

$$n_{uv} = |\{l \in L \text{ s.t. } \chi_E(u, v, l) = 1\}| = \sum_{l \in L} \chi_E(u, v, l)$$

# Applications



- 1 = Wearing
- 2 = Caused
- 3 = AIDS
- 4 = Prevent
- 5 = Elle
- 6 = Macpherson
- 7 = Top

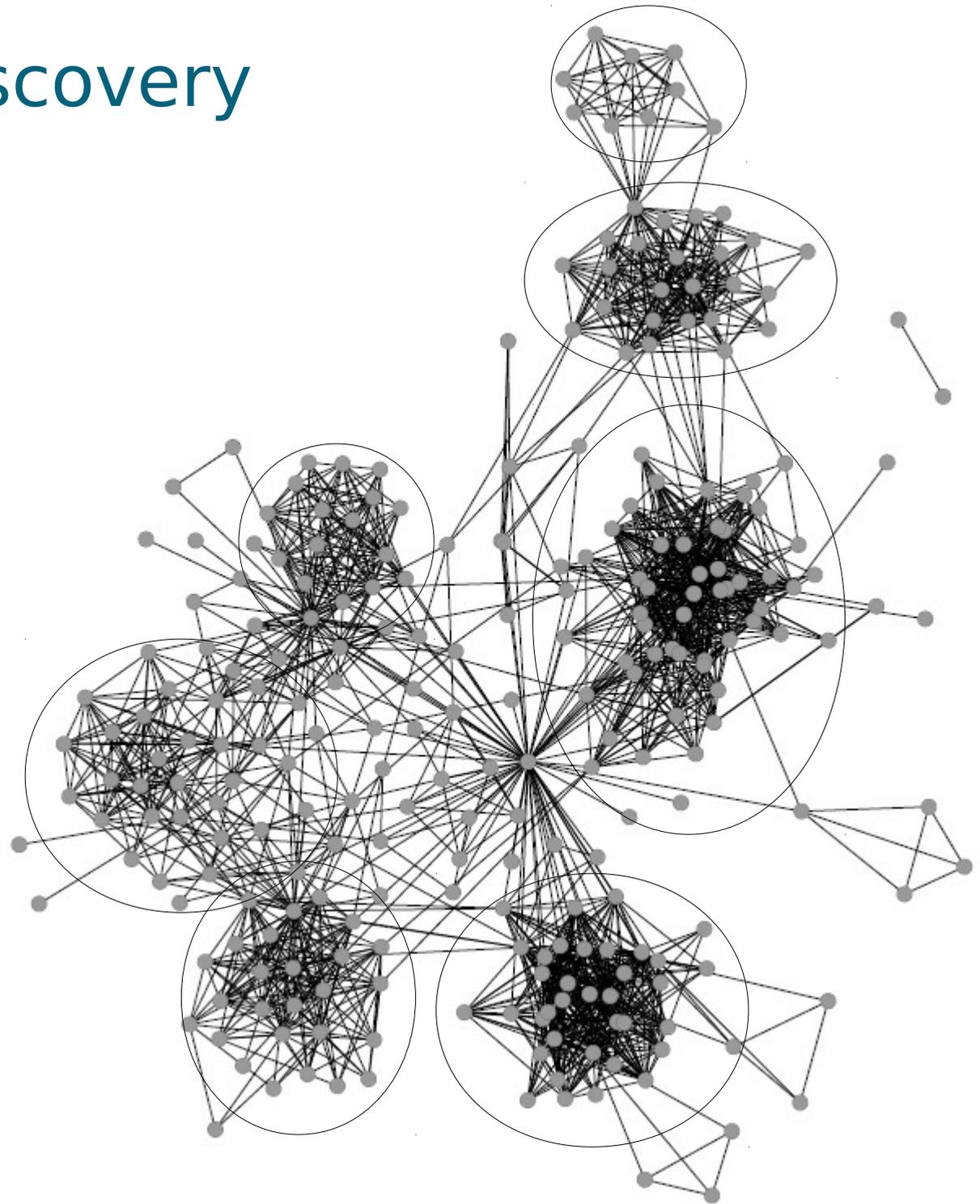


# Community Discovery

Groups of nodes

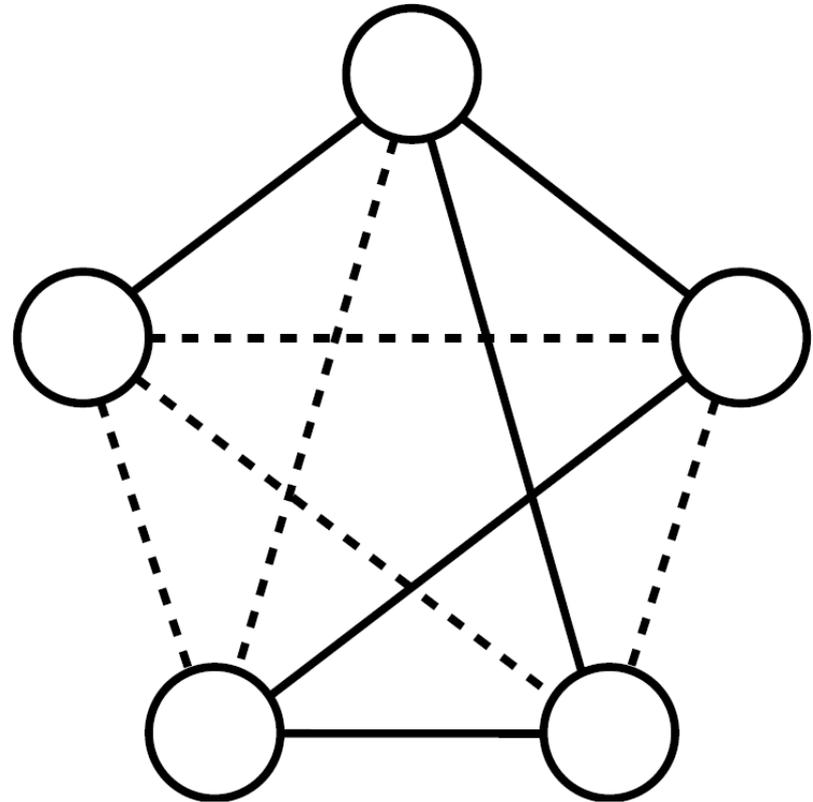
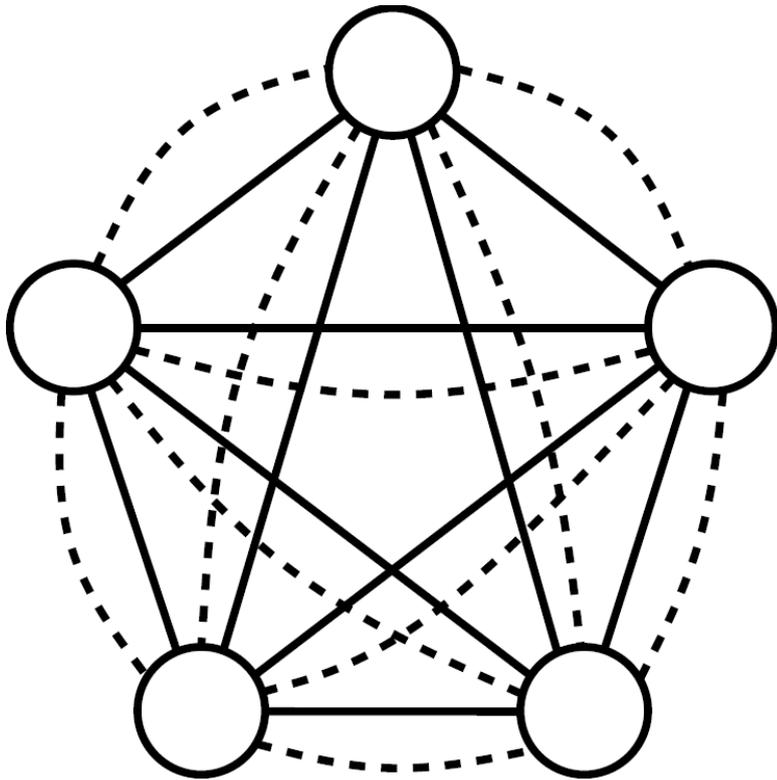
**densely**

connected

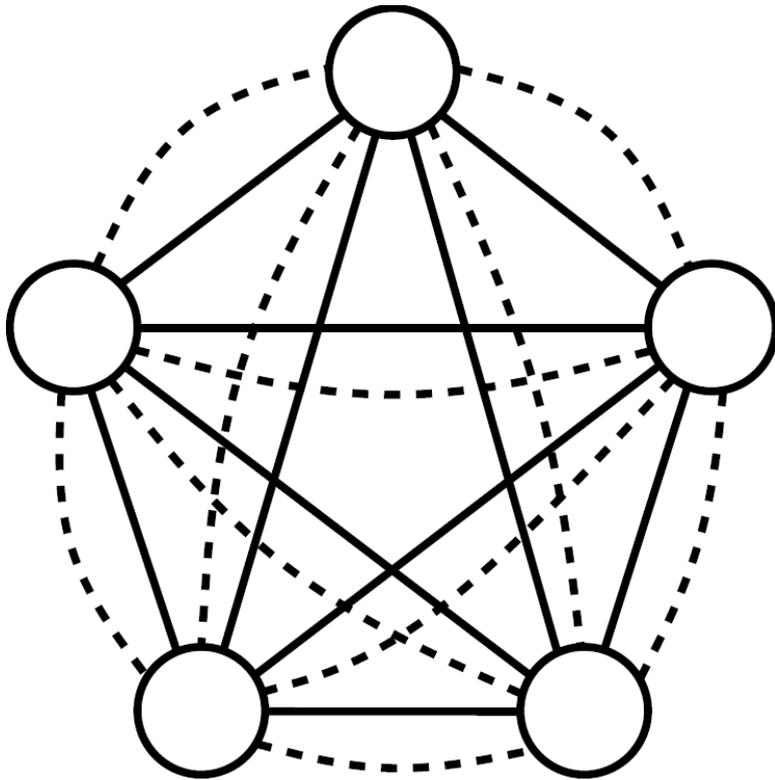


What does  
**multidimensionally densely**  
connected mean?

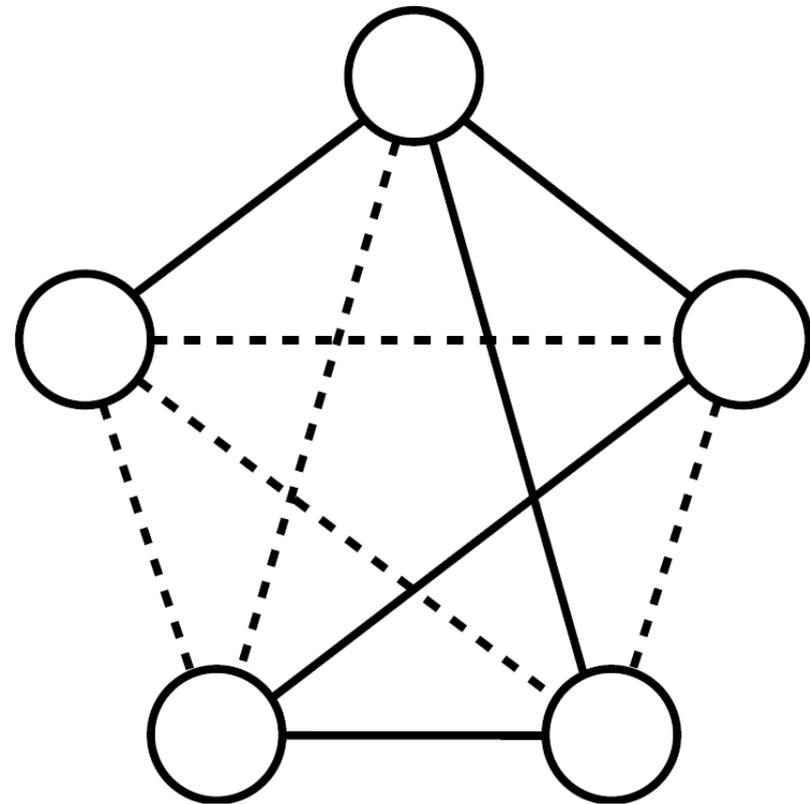
# Two different concepts of “Multidimensional Density”



# Characterizing these concepts



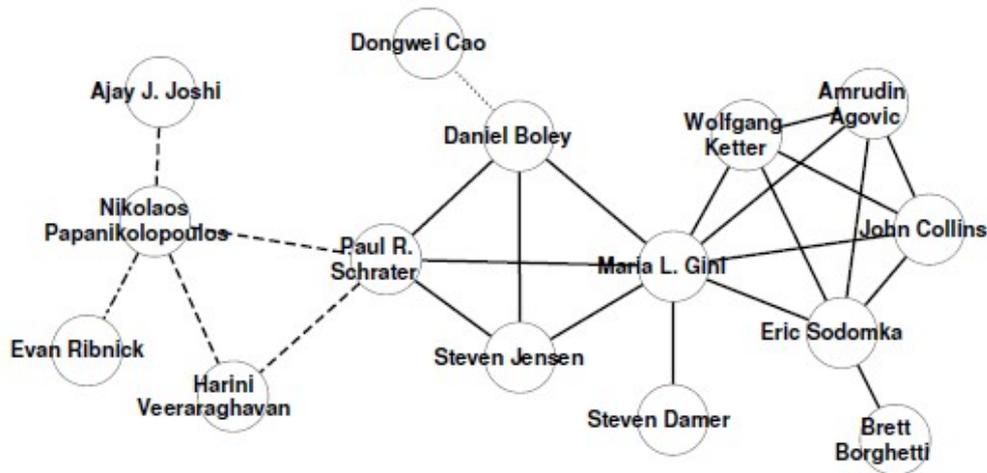
$$\rho_c = \sum_{(u,v) \in \overline{P_c}} \frac{|\{d : \exists (u,v,d) \in E\}|}{|D| \times |P_c|}$$



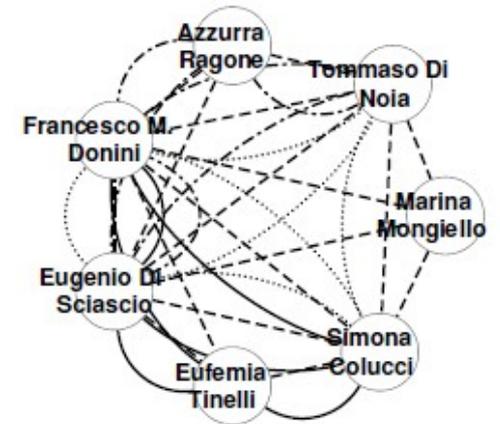
$$\gamma_c = \nu_c \times \varepsilon_c \times \mathcal{H}_c$$

$$\nu_c = \frac{|D_c| - 1}{|D| - 1} \quad \varepsilon_c = \frac{\sum_{d \in D} |\overline{P_{c,d}}|}{|P_c|} \quad \mathcal{H}_c = 1 - \frac{\sigma_c}{\sigma_c^{max}}$$

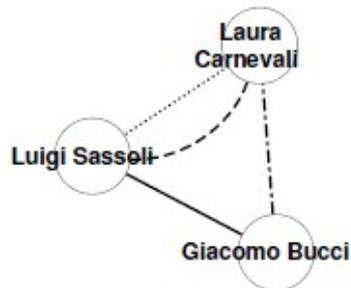
# Usefulness in real world datasets



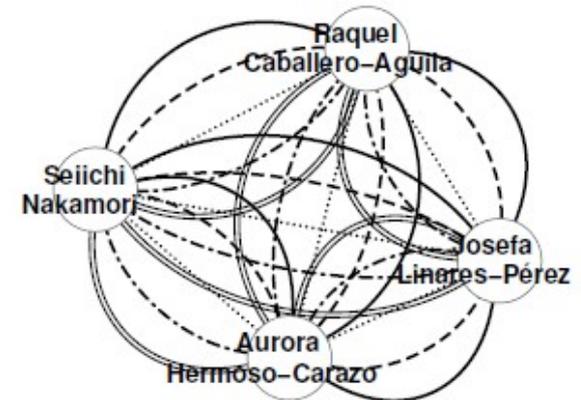
(c) high  $\gamma$  (0.00064) in DBLP-C



(d) high  $\rho$  (0.06048) in DBLP-C



(e) high  $\gamma$  (0.5) in DBLP-Y



(f) high  $\rho$  (1) in DBLP-Y

# Not only Community Discovery

- Link prediction (in which dimension?)
- Shortest path (through which medium? How many hops to/from media?)
- Network models (do dimensions influence each other?)
- Temporal analysis (using snapshots as dimensions)